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The boundary states and correlation functions of the tricritical Ising model from the Coulomb-gas formalism.

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Abstract

We consider the minimal model describing the tricritical Ising model on the upper half plane and using the coulomb-gas formalism we determine its consistents boundary states as well as its 1-point and 2-point correlation functions.

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1 Introduction

To construct boundary conformal field theories (BCFT) one have to know how to maintain the conformal invariance in the desired bounded geometry. This fact is mathematically equivalent to write the equations expressing the contraints imposed by the presence of boudaries on the conformal algebra , on the operator contents of the theory, on the structure of its Hilbert space, on the partition functions and on the correlation functions of the theory.

By imposing the conformal invariance of a given theory defined on a domain with boundaries one obtains the different boundary conditions and the consistent boundary states. From the other part if one considers the equivalence of the partition functions on conformally equivalent geometries

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one obtains a classification of the conformal field theories with boundaries (for review see [7], [8],[9],[10] and [11]).

The basic concepts and techniques of BCFT were introduced first by Cardy. He studied in [1] how to restrict the operator content by imposing the boudary conditions. He gave a classification of the boundary states and he introduced the concept of boundary operators in [2] , [3] and [4].

Later a growing interest on BCFT was motivated by their relation with the D-Branes in string theory (see for example [5] and the references therein).

In our present work, we study the minimal conformal model (A_4, A_3) on the upper half plane using the Coulomb-gas formalism. This model is known to describe the tricritical Ising model [12]. The supersymmetry in the boundary tricritical Ising field theory is studied in [13],[14].

The paper is organised as follows. In section 2, we start by considering the general case of a conformal theory defined on the upper half plane and study the consequences of the existance of boundaries on the conformal algebra and on the structure of its Hilbert space. In section 3, using the Cardy's fusion method [2] we derive a classification of the conformal theories one can define on the bounded geometry in terms of the boundary conditions . In section 4 we determine the consistent boudary states as well as the partition functions for the particular minimal model (A_4, A_3).

In section 5, we give the essential of the method used in [16] and [17] to determine the boundary correlation functions. It consists in writing a free field representation of the consistents boundary states and using the well known coulomb-gas formalism . Then we apply it to determine the one-point as well as the two-point correlators of the tricritical Ising model with boundaries.

2 A CFT on the Upper Half plane

Let us start with a CFT on the upper half plane (UHP). We define $z = x + iy$ and we consider a theory defined on the region $\mathrm{Im}(z) \geq 0$.

Only real analytic changes of coordinates, with

$$\varepsilon(z) = \overline{\varepsilon}(\overline{z})|_{z=\overline{z}\in R} \tag{1}$$

are allowed. The energy momentum verify

$$T(z) = \overline{T}(\overline{z})|_{\text{real axis}} \tag{2}$$

so that there is no momentum flow across the boundary. There is thus only one copy of the virasoro algebra

$$\overline{L}_{-n} = L_n|_{z=\overline{z}} \tag{3}$$

Boundary conditions labelled by α and β (for $\mathrm{Re}(z) \prec 0$ and $\mathrm{Re}(z) \succ 0$) are assigned to fields on the boundary. The UHP can conformally mapped on an infinite horizontal strip of widh L by $w = \frac{L}{\pi} \ln z$

In both geometries, the system is discribed by a Hilbert space of states $\mathcal{H}_{\alpha|\beta}$ which decomposes on representations V_i of the virasoro algebra according to

$$\mathcal{H}_{\alpha|\beta} = \oplus \; {n_{i\alpha}}^{\beta} V_i \tag{4}$$

where the integers ${n_{i\alpha}}^{\beta}$ are the multiplicities of the representations.

On the strip, the Hamiltonien is the translation operator in the direction $\mathrm{Re}(w)$. It can be written in the UHP as

$$H_{\alpha\beta} = \frac{2\pi}{L}(L_0 - \frac{c}{24}) \tag{5}$$

To determine the operator content of the theory, we need to determine the possible boundary conditions α, β on the UHP and also the associated multiplicities ${n_{i\alpha}}^{\beta}$

3 The physical boundary states and the partition functions

We proceed as in the case of the theories defined on the tore. We take a semi annular domain in the half plane and identify its edges to make a cylinder. Equivalently one can consider instide a finite strip. In fact the semi annular domain comprised between the semi-circles of radius 1 and $e^{\frac{\pi T}{L}}$ (in the z UHP) can be mapped in the w plane into the segment $0 \leq \mathrm{Re}(w) \leq T$ of the strip of width L by the transformation $w(z) = \frac{L}{\pi} \ln(z)$. In the same way the latter can also be transformed into the annulus of radius 1 and $e^{\frac{2\pi L}{T}} = \widetilde{q}^{-1/2}$ by the transformation $\zeta(w) = \exp(-2i\pi w/T)$ (with $\widetilde{q} = e^{2\pi i \widetilde{\tau}}, \widetilde{\tau} = 2iT/L$).

The annulus in the ζ plane is equivalent to a finite cylinder of length T and circumference L. When considering such geometry one is allowed to use the familiar energy momentum tensor of the full plane without modification. Then radial quantization is allowed and the conformal invariance condition (2) on the quantum states $|\alpha\rangle$ is [2]

$$(L_n^{\zeta} - \overline{L}_{-n}^{\zeta})\,|\alpha\rangle = 0 \tag{6}$$

There is a basis of states, which are solution of this linear system of boundary conditions. It is the basis of Ishibashi states [6]. There exists an independent Ishibashi states (noted $|j\rangle\rangle$) solution of (6) for each representation V_j of the algebra. The equation (6) being linear, any linear combinaison of the Ishibashi states is also a solution. To obtain the combinaisons corresponding to the physical boundary states, one uses the Cardy's fusion method [2] (for review see also [7], [8],[9],[10] and [11])

It consists in calculating the partition function $\mathcal{Z}_{\alpha|\beta}$ in two different ways. First as resulting from the evolution between the boundary states $\langle\alpha|$ and $|\beta\rangle$

$$\begin{aligned} \mathcal{Z}_{\alpha|\beta} &= \left\langle \alpha \left| e^{-TH} \right| \beta \right\rangle \\ &= \left\langle \alpha \left| \widetilde{q}^{\frac{1}{2}(L_0+\overline{L}_0-\frac{c}{12})} \right| \beta \right\rangle \end{aligned} \tag{7}$$

where $H = \frac{2\pi}{L}(L_0 + \overline{L}_0 - \frac{c}{12})$ and $\widetilde{q} = \exp(-4\pi\frac{T}{L})$

On the other hand using the decomposition (4) of the hilbert space $\mathcal{H}_{\alpha|\beta}$ one obtain

$$\mathcal{Z}_{\alpha|\beta}(q) = \sum_j n_{j\alpha}^{\ \beta} \chi_j(q) \tag{8}$$

where $\chi_j(q)$ is the caracter of the representation V_j and $q = e^{-\pi L/T}$

Expanding the physical boundary states as

$$|\alpha\rangle = \sum_j \frac{a_{\alpha j}}{\sqrt{S_{1j}}} |j\rangle\rangle \tag{9}$$

S_{ij} being the element of the matrix S of the modular group and equating (7) and (8), we obtain

$$\sum_j n_{j\alpha}^{\ \beta} \chi_j(q) = \sum_{jj'} \langle\alpha|j\rangle\rangle\langle\langle j|\widetilde{q}^{\frac{1}{2}(L_0+\overline{L}_0-\frac{c}{12})}|j'\rangle\rangle\langle\langle j'|\beta\rangle \tag{10}$$

The Ishibashi states are normalized so that

$$\langle\langle j|j'\rangle\rangle = \delta_{jj'} S_{1j} \tag{11}$$

and

$$\langle\langle j|\widetilde{q}^{\frac{1}{2}(L_0+\overline{L}_0-\frac{c}{12})}|j'\rangle\rangle = \delta_{jj'}\chi_j(\widetilde{q}) \tag{12}$$

then (10) becomes

$$\sum_j n_{j\alpha}{}^{\beta}\chi_j(q) = \sum_j \frac{a^*_{\alpha j}a_{\beta j}}{S_{1j}}\chi_j(\widetilde{q}) \tag{13}$$

Now performing a modular transformation on the caracters

$$\chi_j(\widetilde{q}) = \sum_i S_{ij}\chi_i(q) \tag{14}$$

and identifying the coefficients of χ_i, one obtains

$$n_{j\alpha}{}^{\beta} = \sum_i \frac{S_{ji}a^*_{\alpha j}a_{\beta j}}{S_{1i}} \tag{15}$$

To impose the orthonormality condition $\langle\alpha|\beta\rangle = \delta_{\alpha\beta}$, the coefficients $a_{\alpha j}$ have to verify

$$\sum_j a^*_{\alpha j}a_{\beta j} = \delta_{\alpha\beta} \tag{16}$$

As a solution, one can take (recall that the matrix S is unitary)

$$a_{ij} = S_{ij} \tag{17}$$

so that finally we have for the physical (or what we also call consistent) boundary states

$$|\alpha\rangle = \sum_j \frac{S_{\alpha j}}{\sqrt{S_{1j}}}|j\rangle\rangle \tag{18}$$

and for the multiplicities

$$n_{j\alpha}{}^{\beta} = \sum_i \frac{S_{ji}S^*_{\alpha j}S_{\beta j}}{S_{1i}} \tag{19}$$

This equation is no more than the Verlinde formula [15]. It gives the fusion multiplicities apparing into the decomposition of the fusion of two representations of the algebra in terms of the elements of the S matrix.

Thus the multiplicities $n_{i\alpha}{}^{\beta}$ are identified with the coefficients of the fusion algebra

$$n_{i\alpha}{}^{\beta} = N_{i\alpha}{}^{\beta} \tag{20}$$

We obtain then a classification of the different conformal field theories on the upper half plane and their operator contents. The later depends on the boundary conditions.

4 The case of the tricritical ising model (A_4, A_3) :

The central charge, in this case, is $c = 7/10$, and the Kac table contents six primary fields indexed by the pairs (r, s) given in the set ε. See table (1) for operator contents of the model.

$$\varepsilon = \left\{ \begin{array}{c} (1,1) = (3,4), (1,2) = (3,3), (1,3) = (3,2), \\ (1,4) = (3,1), (2,2) = (2,3), (2,4) = (2,1) \end{array} \right\}$$

(r, s)	Dimension	signe
$(1,1)$ or $(3,4)$	0	I
$(1,2)$ or $(3,3)$	$1/10$	ε
$(1,3)$ or $(3,2)$	$3/5$	ε'
$(1,4)$ or $(3,1)$	$3/2$	ε''
$(2,2)$ or $(2,3)$	$3/80$	σ
$(2,4)$ or $(2,1)$	$7/16$	σ'

Table 1: The primary fields of the tricritical Ising model.

The modular matrice S is given by

$$S = \frac{1}{\sqrt{5}} \begin{pmatrix} s_2 & s_1 & s_1 & s_2 & \sqrt{2}s_1 & \sqrt{2}s_2 \\ s_1 & -s_2 & -s_2 & s_1 & \sqrt{2}s_2 & -\sqrt{2}s_1 \\ s_1 & -s_2 & -s_2 & s_1 & -\sqrt{2}s_2 & \sqrt{2}s_1 \\ s_2 & s_1 & s_1 & s_2 & -\sqrt{2}s_1 & -\sqrt{2}s_2 \\ \sqrt{2}s_1 & \sqrt{2}s_2 & -\sqrt{2}s_2 & -\sqrt{2}s_1 & 0 & 0 \\ \sqrt{2}s_2 & -\sqrt{2}s_1 & \sqrt{2}s_1 & -\sqrt{2}s_2 & 0 & 0 \end{pmatrix} \tag{21}$$

with $s_1 = \sin 2\pi/5$, and $s_2 = \sin 4\pi/5$.

4.1 The physical boundary states

As we already mentionned for the diagonal minimal models of type $(A_{p-1}, A_{p'-1})$, the coefficients a^j_α introduced in equation (9) are equal to the elements of the matrix S [8] . Then the Consistent boundary state for this model are writen as

$$|(r,s)\rangle = \sum_{(r',s')\in\varepsilon} \frac{S_{rs,r's'}}{\sqrt{S_{11,r's'}}} |(r',s')\rangle\rangle \tag{22}$$

These are the six states given in the following table

The field operator	The Consistent boundary state
I	$\vert I\rangle=\vert(1,1)\rangle=\frac{1}{\sqrt[4]{5}}\{\sqrt{s_2}\vert I\rangle\rangle+\sqrt{s_1}\vert\varepsilon\rangle\rangle+\sqrt{s_1}\vert\varepsilon'\rangle\rangle+\sqrt{s_2}\vert\varepsilon''\rangle\rangle+\sqrt{\sqrt{2}s_1}\vert\sigma\rangle\rangle+\sqrt{\sqrt{2}s_2}\vert\sigma'\rangle\rangle\}$
ε	$\vert\varepsilon\rangle=\vert(1,2)\rangle=\frac{1}{\sqrt[4]{5}}\{\frac{s_1}{\sqrt{s_2}}\vert I\rangle\rangle-\frac{s_2}{\sqrt{s_1}}\vert\varepsilon\rangle\rangle-\frac{s_2}{\sqrt{s_1}}\vert\varepsilon'\rangle\rangle+\frac{s_1}{\sqrt{s_2}}\vert\varepsilon''\rangle\rangle+\sqrt[4]{2}\frac{s_2}{\sqrt{s_1}}\vert\sigma\rangle\rangle-\sqrt[4]{2}\frac{s_1}{\sqrt{s_2}}\vert\sigma'\rangle\rangle\}$
ε'	$\vert\varepsilon'\rangle=\vert(1,3)\rangle=\frac{1}{\sqrt[4]{5}}\{\frac{s_1}{\sqrt{s_2}}\vert I\rangle\rangle-\frac{s_2}{\sqrt{s_1}}\vert\varepsilon\rangle\rangle-\frac{s_2}{\sqrt{s_1}}\vert\varepsilon'\rangle\rangle+\frac{s_1}{\sqrt{s_2}}\vert\varepsilon''\rangle\rangle-\sqrt[4]{2}\frac{s_2}{\sqrt{s_1}}\vert\sigma\rangle\rangle+\sqrt[4]{2}\frac{s_1}{\sqrt{s_2}}\vert\sigma'\rangle\rangle$
ε''	$\vert\varepsilon''\rangle=\vert(1,4)\rangle=\frac{1}{\sqrt[4]{5}}\{\sqrt{s_2}\vert I\rangle\rangle+\sqrt{s_1}\vert\varepsilon\rangle\rangle+\sqrt{s_1}\vert\varepsilon'\rangle\rangle+\sqrt{s_2}\vert\varepsilon''\rangle\rangle-\sqrt{\sqrt{2}s_1}\vert\sigma\rangle\rangle-\sqrt{\sqrt{2}s_2}\vert\sigma'\rangle\rangle\}$
σ	$\vert\sigma\rangle=\vert(2,2)\rangle=\frac{1}{\sqrt[4]{5}}\{\sqrt{2}\frac{s_1}{\sqrt{s_2}}\vert I\rangle\rangle+\sqrt{2}\frac{s_2}{\sqrt{s_1}}\vert\varepsilon\rangle\rangle-\sqrt{2}\frac{s_2}{\sqrt{s_1}}\vert\varepsilon'\rangle\rangle-\sqrt{2}\times\frac{s_1}{\sqrt{s_2}}\vert\varepsilon''\rangle\rangle\}$
σ'	$\vert\sigma'\rangle=\vert(2,4)\rangle=\frac{1}{\sqrt[4]{5}}\{\sqrt{2s_2}\vert I\rangle\rangle-\sqrt{2s_1}\vert\varepsilon\rangle\rangle+\sqrt{2s_1}\vert\varepsilon'\rangle\rangle-\sqrt{2s_2}\vert\varepsilon''\rangle\rangle\}$

Table 2: The Consistent boundary states of the tricritical Ising model

4.2 The fusion rules and the partition functions

The fusion between the differents operators of the model are resumed as follows

$$\begin{pmatrix} I & \varepsilon & \varepsilon' & \varepsilon'' & \sigma & \sigma' \\ \varepsilon & \varepsilon^2 & \varepsilon\varepsilon' & \varepsilon\varepsilon'' & \varepsilon\sigma & \varepsilon\sigma' \\ \varepsilon' & \varepsilon'\varepsilon & \varepsilon'^{\,2} & \varepsilon'\varepsilon'' & \varepsilon'\sigma & \varepsilon'\sigma' \\ \varepsilon'' & \varepsilon''\varepsilon & \varepsilon''\varepsilon' & \varepsilon''^{\,2} & \varepsilon''\sigma & \varepsilon''\sigma' \\ \sigma & \sigma\varepsilon & \sigma\varepsilon' & \sigma\varepsilon'' & \sigma^2 & \sigma\sigma' \\ \sigma' & \sigma'\varepsilon & \sigma'\varepsilon' & \sigma'\varepsilon'' & \sigma'\sigma & \sigma'^{\,2} \end{pmatrix} =$$

$$= \begin{pmatrix} 1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1 \end{pmatrix} I + \begin{pmatrix} 0&1&0&0&0&0\\1&0&1&0&0&0\\0&1&0&1&0&0\\0&0&1&0&0&0\\0&0&0&0&1&1\\0&0&0&0&1&0 \end{pmatrix} \varepsilon$$

$$+ \begin{pmatrix} 0&0&1&0&0&0\\0&1&0&1&0&0\\1&0&1&0&0&0\\0&1&0&0&0&0\\0&0&0&0&1&1\\0&0&0&0&1&0 \end{pmatrix} \varepsilon' + \begin{pmatrix} 0&0&0&1&0&0\\0&0&1&0&0&0\\0&1&0&0&0&0\\1&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1 \end{pmatrix} \varepsilon''$$

$$+\begin{pmatrix} 0&0&0&0&1&0\\ 0&0&0&0&1&1\\ 0&0&0&0&1&1\\ 0&0&0&0&1&0\\ 1&1&1&1&0&0\\ 0&1&1&0&0&0 \end{pmatrix}\sigma+\begin{pmatrix} 0&0&0&0&0&1\\ 0&0&0&0&1&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 0&1&1&0&0&0\\ 1&0&0&1&0&0 \end{pmatrix}\sigma' \tag{23}$$

Using the equations (8) and (20) we obtain the twelve following partition functions

$$\begin{aligned}
\mathcal{Z}_{I|I} &= \mathcal{Z}_{\varepsilon''|\varepsilon''} = \chi_{1,1}(q),\\
\mathcal{Z}_{I|\varepsilon} &= \mathcal{Z}_{\varepsilon'|\varepsilon''} = \chi_{1,2}(q),\\
\mathcal{Z}_{I|\varepsilon'} &= \mathcal{Z}_{\varepsilon|\varepsilon''} = \chi_{1,3}(q),\\
\mathcal{Z}_{I|\varepsilon''} &= \chi_{1,4}(q),\\
\mathcal{Z}_{I|\sigma} &= \mathcal{Z}_{\varepsilon|\sigma'} = \mathcal{Z}_{\varepsilon'|\sigma'} = \mathcal{Z}_{\varepsilon''|\sigma} = \chi_{2,2}(q),\\
\mathcal{Z}_{I|\sigma'} &= \mathcal{Z}_{\varepsilon''|\sigma'} = \chi_{2,4}(q),\\
\mathcal{Z}_{\varepsilon|\varepsilon} &= \mathcal{Z}_{\varepsilon'|\varepsilon'} = \chi_{1,1}(q)+\chi_{1,3}(q),\\
\mathcal{Z}_{\varepsilon|\varepsilon'} &= \chi_{1,2}(q)+\chi_{1,4}(q),\\
\mathcal{Z}_{\varepsilon|\sigma} &= \mathcal{Z}_{\varepsilon'|\sigma} = \chi_{2,2}(q)+\chi_{2,4}(q),\\
\mathcal{Z}_{\sigma|\sigma} &= \chi_{1,1}(q)+\chi_{1,2}(q)+\chi_{1,3}(q)+\chi_{1,4}(q),\\
\mathcal{Z}_{\sigma|\sigma'} &= \chi_{1,2}(q)+\chi_{1,3}(q),\\
\mathcal{Z}_{\sigma'|\sigma'} &= \chi_{1,1}(q)+\chi_{1,4}(q)
\end{aligned} \tag{24}$$

where we have taken into account the fact that

$$\mathcal{Z}_{\alpha|\beta} = \mathcal{Z}_{\beta|\alpha} \tag{25}$$

5 The boundary correlation functions

In this section we present the formalism developed in [16] and [17] for calculating boundary correlation functions. This method is based on the coulomb-gas picture and uses the free-field representation of the boundary states. We will first recall with few details the most important steps of this method.

5.1 The coulomb gas formalism and the coherent boundary states

In the coulomb gas-formalism one reproduces the primary fields $\phi_{r,s}$ ($0 < r < p, 0 < s < p'$) of the Kac table of the minimal model $(A_{p-1}, A_{p'-1})$ by the vertex operators

$$\mathcal{V}_{\alpha_{r,s}}(z) =: \exp(i\sqrt{2}\alpha_{r,s}\varphi(z) : \tag{26}$$

with conformal dimensions given by

$$h_{r,s} = \frac{1}{4}(r\alpha_+ + s\alpha_-)^2 - \alpha_0^2 \tag{27}$$

where $\varphi(z)$ is the holomorphic part of a boson field $\Phi(z, \overline{z})$ and

$$\begin{aligned} \alpha_{r,s} &= \frac{1}{2}(1-r)\alpha_+ + \frac{1}{2}(1-s)\alpha_- \\ \alpha_+ &= \sqrt{p/p'} \\ \alpha_- &= -\sqrt{p'/p} \end{aligned} \tag{28}$$

The coefficient α_0 is related to the central charge by

$$c = 1 - 24\alpha_0^2 \tag{29}$$

It is known (see for example [18]) that the correlation function of n vertex operators is equal to

$$\left\langle \prod_{i=1}^{n} \mathcal{V}_{\alpha_i}(z_i) \right\rangle = \left\{ \begin{array}{ccc} \prod_{i<j}^{n}(z_i - z_j)^{2\alpha_i\alpha_j} & \text{if} & \sum_{i=1}^{n}\alpha_i = 2\alpha_0 \\ 0 & & \text{otherwise} \end{array} \right\} \tag{30}$$

The fock space F_{α,α_0} related to the vertex operator $\mathcal{V}_\alpha$ is built on the highest weight state

$$|\alpha, \alpha_0\rangle = \exp(i\sqrt{2}\alpha\widehat{\varphi}_0)\,|0, \alpha_0\rangle$$

by the action of the mode operators $\widehat{a}_m$ of the field $\varphi(z)$, defined by

$$\varphi(z) = \widehat{\varphi_0} - i\widehat{a}_0 \ln(z) + i\sum_{n\neq 0} \frac{\widehat{a}_n}{n} z^{-n} \tag{31}$$

and satisfying the algebra

$$\begin{aligned} [\widehat{a}_m, \widehat{a}_n] &= m\delta_{m+n} \\ [\widehat{\varphi}_0, \widehat{a}_0] &= i \end{aligned} \tag{32}$$

The state $|0, \alpha_0\rangle$ is the vacuum state and we have

$$\widehat{a}_0\,|\alpha, \alpha_0\rangle = \sqrt{2}\alpha\,|\alpha, \alpha_0\rangle$$

$$\langle \alpha, \alpha_0 | \beta, \alpha_0 \rangle = \kappa \delta_{\alpha\beta}$$

κ being a normalization constant which can be choosen to be equal to one.

The aim goal now is to construct conformally boundary states in terms of the states $|\alpha, \alpha_0\rangle$ of the different fock spaces corresponding to the different values of α. Starting from the ansatz

$$|B_{\alpha,\overline{\alpha},\alpha_0}\rangle = \prod_{k>0} \exp(-\frac{1}{k} \widehat{a}_{-k} \widehat{\overline{a}}_{-k}) \, |\alpha, \overline{\alpha}, \alpha_0\rangle \tag{33}$$

where the states $|\alpha, \overline{\alpha}, \alpha_0\rangle$ are the direct product of the holomorphic and antiholomorphic highest weight states

$$|\alpha, \overline{\alpha}, \alpha_0\rangle = |\alpha, \alpha_0\rangle \otimes |\overline{\alpha}, \alpha_0\rangle \tag{34}$$

and by expressing the elements of the virasoro algebra in terms of the operators $\widehat{a}_m$, one can easily show that the states $|B_{\alpha,\overline{\alpha},\alpha_0}\rangle$ verify

$$(L_n - \overline{L}_{-n}) \, |B_{\alpha,\overline{\alpha},\alpha_0}\rangle = 0 \tag{35}$$

only if

$$\alpha + \overline{\alpha} - 2\alpha_0 = 0 \tag{36}$$

The boundary states satisfying these two conditions will be noted

$$|B_{\alpha,\overline{\alpha},\alpha_0}\rangle = |B_{\alpha,2\alpha_0-\alpha,\alpha_0}\rangle = |B(\alpha)\rangle \tag{37}$$

In addition to the necessary condition (35), the coherent states $|B(\alpha)\rangle$ have to satisfy the conditions obtained from the duality of the partition function as already done in section 1. There is in fact a combinaison of these states which satisfy the duality condition and it is shown in [16] that the Ishibashi states are related to these states as

$$|(r,s) >>= |a_{r,s} > + |a_{r,-s} > \tag{38}$$

with

$$|a_{r,s} >= \sum_{k \in Z} |B(k\sqrt{pp'} + \alpha_{r,s}) > \tag{39}$$

Using (38) and (18) one can write the physical (or consistent) boundary states of any minimal model in terms of the coherent ones. For the particular case of the minimal model (A_4, A_3) we have the following relations

$$|I\rangle = \tfrac{1}{\sqrt{2}} \tfrac{1}{\sqrt[4]{5}} \{ \sqrt{s_2}(|a_{1,1}\rangle + |a_{1,-1}\rangle + |a_{3,4}\rangle + |a_{3,-4}\rangle) + \sqrt{s_1}(|a_{1,2}\rangle + |a_{1,-2}\rangle + |a_{3,3}\rangle + |a_{3,-3}\rangle) + \sqrt{s_1}(|a_{1,3}\rangle + |a_{1,-3}\rangle + |a_{3,2}\rangle + |a_{3,-2}\rangle) +$$

$$
\begin{aligned}
&\sqrt{s_2}(|a_{1,4}\rangle+|a_{1,-4}\rangle+|a_{3,1}\rangle+|a_{3,-1}\rangle)+\sqrt{\sqrt{2}s_1}(|a_{2,2}\rangle+|a_{2,-2}\rangle+\\
&|a_{2,3}\rangle+|a_{2,-3}\rangle)+\sqrt{\sqrt{2}s_2}(|a_{2,4}\rangle+|a_{2,-4}\rangle+|a_{2,1}\rangle+|a_{2,-1}\rangle)\}\\
|\varepsilon\rangle=&\tfrac{1}{\sqrt{2}}\tfrac{1}{\sqrt[4]{5}}\{\tfrac{s_1}{\sqrt{s_2}}(|a_{1,1}\rangle+|a_{1,-1}\rangle+|a_{3,4}\rangle+|a_{3,-4}\rangle)-\tfrac{s_2}{\sqrt{s_1}}(|a_{1,2}\rangle+\\
&|a_{1,-2}\rangle+|a_{3,3}\rangle+|a_{3,-3}\rangle)-\tfrac{s_2}{\sqrt{s_1}}(|a_{1,3}\rangle+|a_{1,-3}\rangle+|a_{3,2}\rangle+|a_{3,-2}\rangle)+\\
&\tfrac{s_1}{\sqrt{s_2}}(|a_{1,4}\rangle+|a_{1,-4}\rangle+|a_{3,1}\rangle+|a_{3,-1}\rangle)+\sqrt[4]{2}\tfrac{s_2}{\sqrt{s_1}}(|a_{2,2}\rangle+|a_{2,-2}\rangle+\\
&|a_{2,3}\rangle+|a_{2,-3}\rangle)-\sqrt[4]{2}\tfrac{s_1}{\sqrt{s_2}}(|a_{2,4}\rangle+|a_{2,-4}\rangle+|a_{2,1}\rangle+|a_{2,-1}\rangle)\}\\
|\varepsilon'\rangle=&\tfrac{1}{\sqrt{2}}\tfrac{1}{\sqrt[4]{5}}\{\tfrac{s_1}{\sqrt{s_2}}(|a_{1,1}\rangle+|a_{1,-1}\rangle+|a_{3,4}\rangle+|a_{3,-4}\rangle)-\tfrac{s_2}{\sqrt{s_1}}(|a_{1,2}\rangle+\\
&|a_{1,-2}\rangle+|a_{3,3}\rangle+|a_{3,-3}\rangle)-\tfrac{s_2}{\sqrt{s_1}}(|a_{1,3}\rangle+|a_{1,-3}\rangle+|a_{3,2}\rangle+|a_{3,-2}\rangle)+\\
&\tfrac{s_1}{\sqrt{s_2}}(|a_{1,4}\rangle+|a_{1,-4}\rangle+|a_{3,1}\rangle+|a_{3,-1}\rangle)-\sqrt[4]{2}\tfrac{s_2}{\sqrt{s_1}}(|a_{2,2}\rangle+|a_{2,-2}\rangle+\\
&|a_{2,3}\rangle+|a_{2,-3}\rangle)+\sqrt[4]{2}\tfrac{s_1}{\sqrt{s_2}}(|a_{2,4}\rangle+|a_{2,-4}\rangle+|a_{2,1}\rangle+|a_{2,-1}\rangle)\}\\
|\varepsilon''\rangle=&\tfrac{1}{\sqrt{2}}\tfrac{1}{\sqrt[4]{5}}\{\sqrt{s_2}(|a_{1,1}\rangle+|a_{1,-1}\rangle+|a_{3,4}\rangle+|a_{3,-4}\rangle)+\sqrt{s_1}(|a_{1,2}\rangle+\\
&|a_{1,-2}\rangle+|a_{3,3}\rangle+|a_{3,-3}\rangle)+\sqrt{s_1}(|a_{1,3}\rangle+|a_{1,-3}\rangle+|a_{3,2}\rangle+|a_{3,-2}\rangle)+\\
&\sqrt{s_2}(|a_{1,4}\rangle+|a_{1,-4}\rangle+|a_{3,1}\rangle+|a_{3,-1}\rangle)-\sqrt{\sqrt{2}s_1}(|a_{2,2}\rangle+|a_{2,-2}\rangle+\\
&|a_{2,3}\rangle+|a_{2,-3}\rangle)-\sqrt{\sqrt{2}s_2}(|a_{2,4}\rangle+|a_{2,-4}\rangle+|a_{2,1}\rangle+|a_{2,-1}\rangle)\}\\
|\sigma\rangle=&\tfrac{1}{\sqrt[4]{5}}\{\tfrac{s_1}{\sqrt{s_2}}(|a_{1,1}\rangle+|a_{1,-1}\rangle+|a_{3,4}\rangle+|a_{3,-4}\rangle)+\tfrac{s_2}{\sqrt{s_1}}(|a_{1,2}\rangle+\\
&|a_{1,-2}\rangle+|a_{3,3}\rangle+|a_{3,-3}\rangle)-\tfrac{s_2}{\sqrt{s_1}}(|a_{1,3}\rangle+|a_{1,-3}\rangle+|a_{3,2}\rangle+|a_{3,-2}\rangle)-\\
&\tfrac{s_1}{\sqrt{s_2}}(|a_{1,4}\rangle+|a_{1,-4}\rangle+|a_{3,1}\rangle+|a_{3,-1}\rangle)\\
|\sigma'\rangle=&\tfrac{1}{\sqrt[4]{5}}\{\sqrt{s_2}(|a_{1,1}\rangle+|a_{1,-1}\rangle+|a_{3,4}\rangle+|a_{3,-4}\rangle)-\sqrt{s_1}(|a_{1,2}\rangle+\\
&|a_{1,-2}\rangle+|a_{3,3}\rangle+|a_{3,-3}\rangle)+\sqrt{s_1}(|a_{1,3}\rangle+|a_{1,-3}\rangle+|a_{3,2}\rangle+|a_{3,-2}\rangle)-\\
&\sqrt{s_2}(|a_{1,4}\rangle+|a_{1,-4}\rangle+|a_{3,1}\rangle+|a_{3,-1}\rangle)
\end{aligned}
\tag{40}
$$

Note that before using (38), we have replaced each state $|(r,s)\rangle\rangle$ by the symmetrized one

$$\frac{1}{\sqrt{2}}\{|(r,s)\rangle\rangle+|(p'-r,p-s)\rangle\rangle\}$$

5.2 The boundary 1-point correlation functions

In the coulomb gas formalism a boundary p-points correlation function of the form

$$\langle\alpha|\,\phi_{r_1,s_1}(z_1,\overline{z}_1)\phi_{r_2,s_2}(z_2,\overline{z}_2)......\phi_{r_p,s_p}(z_p,\overline{z}_p)\,|0\rangle$$

where $|\alpha\rangle$ is one of the allowed physical boundary state for the model, can be written as a combinaison of correlators having the general form

$$\langle B(\alpha)|\prod_{i=1}^{p}\mathcal{V}^{(m_i,n_i),(\overline{m}_i,\overline{n}_i)}_{(r_i,s_i),(\overline{r}_i,\overline{s}_i)}(z_i,\overline{z}_i)\,|0,0;\alpha_0\rangle \tag{41}$$

Where we have noted

$$\mathcal{V}^{(m_i,n_i),(\overline{m}_i,\overline{n}_i)}_{(r_i,s_i),(\overline{r}_i,\overline{s}_i)}(z_i,\overline{z}_i) = \mathcal{V}^{(m_i,n_i)}_{(r_i,s_i)}(z_i)\overline{\mathcal{V}}^{(\overline{m}_i,\overline{n}_i)}_{(\overline{r}_i,\overline{s}_i)}(\overline{z}_i) \tag{42}$$

with the screened vertex operators

$$\begin{aligned}
\mathcal{V}^{(m,n)}_{(r,s)}(z) &= \oint \prod_{i=1}^{m} du_i \prod_{j=1}^{n} dv_j \mathcal{V}_{r,s}(z)\mathcal{V}_+(u_1)...\mathcal{V}_+(u_m) \times \mathcal{V}_-(v_1)...\mathcal{V}_-(v_n), \\
\overline{\mathcal{V}}^{(\overline{m},\overline{n})}_{(\overline{r},\overline{s})}(\overline{z}) &= \oint \prod_{i=1}^{\overline{m}} d\overline{u}_i \prod_{j=1}^{\overline{n}} d\overline{v}_j \overline{\mathcal{V}}_{\overline{r},\overline{s}}(\overline{z})\mathcal{V}_+(\overline{u}_1)...\mathcal{V}_+(\overline{u}_{\overline{m}}) \times \mathcal{V}_-(\overline{v}_1)...\mathcal{V}_-(\overline{v}_{\overline{n}})
\end{aligned} \tag{43}$$

We recall that the numbers m and n of the screening operators

$$Q_\pm = \oint dz \mathcal{V}_{\alpha_\pm}(z) = \oint dz \mathcal{V}_\pm(z)$$

(of conformal dimensions 1) is so that the neutrality condition appearing in (30) is fullfiled. For the correlator (41) the neutrality condition for the holomorphic part is

$$-\alpha + \sum_i \alpha_{r_i,s_i} + \sum_i m_i\alpha_+ \sum_i n_i\alpha_- = 0 \tag{44}$$

and for the antiholomortphic one, we have

$$\alpha - 2\alpha_0 + \sum_i \alpha_{\overline{r}_i,\overline{s}_i} + \sum_i \overline{m}_i\alpha_+ \sum_i \overline{n}_i\alpha_- = 0 \tag{45}$$

For example for the 1-point correlation function

$$\langle B(\alpha)|\, \phi_{(r,s;\overline{r},\overline{s})}(z,\overline{z})\, |0,0;\alpha_0\rangle = \langle B(\alpha)|\, \mathcal{V}^{(m,n)}_{(r,s)}(z)\overline{\mathcal{V}}^{(\overline{m},\overline{n})}_{(r,s)}(\overline{z})\, |0,0;\alpha_0\rangle \tag{46}$$

with $(\overline{r},\overline{s}) = (r,s)$. The combinaison of the equations (44) and (45) leads to

$$m = \overline{m} = n = \overline{n} = 0 \tag{47}$$

So that in this case there is no screening operators and one find

$$\alpha = \alpha_{r,s} \tag{48}$$

Then we have

$$\begin{aligned}\langle B(\alpha_{r,s})|\,\Phi_{(r,s;\overline{r},\overline{s})}(z,\overline{z})\,|0,0;\alpha_0\rangle &= \langle B(\alpha_{r,s})|\,\mathcal{V}_{(r,s)}(z)\overline{\mathcal{V}}_{(\overline{r},\overline{s})}(\overline{z})\,|0,0;\alpha_0\rangle \\ &= (1-z\overline{z})^{-2h_{r,s}}\end{aligned} \tag{49}$$

Applying these alghorithm for the Tricritical Ising model defined on the unit disk (UD) (i.e in the $\zeta - plane$) one obtains

$$\begin{aligned}\left\langle I\left|I(\zeta,\overline{\zeta})\right|0\right\rangle &= \left\langle I\left|\mathcal{V}_{1,1}(\zeta)\overline{\mathcal{V}}_{3,4}(\overline{\zeta})\right|0,0;\alpha_0\right\rangle \\ &= \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_2}{2}}\,\langle B(\alpha_{1,1})|\,\mathcal{V}_{1,1}(\zeta)\overline{\mathcal{V}}_{3,4}(\overline{\zeta})\,|0,0;\alpha_0\rangle \\ &= \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_2}{2}}(1-\zeta\overline{\zeta})^{-2h_{1,1}} = \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_2}{2}}, \\ \left\langle \varepsilon\left|I(\zeta,\overline{\zeta})\right|0\right\rangle &= \langle\varepsilon|0\rangle = \frac{1}{\sqrt[4]{5}}\frac{s_1}{\sqrt{2s_2}}, \\ \left\langle \varepsilon'\left|I(\zeta,\overline{\zeta})\right|0\right\rangle &= \langle\varepsilon'|0\rangle = \frac{1}{\sqrt[4]{5}}\frac{s_1}{\sqrt{2s_2}}, \\ \left\langle \varepsilon''\left|I(\zeta,\overline{\zeta})\right|0\right\rangle &= \langle\varepsilon''|0\rangle = \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_2}{2}}, \\ \left\langle \sigma\left|I(\zeta,\overline{\zeta})\right|0\right\rangle &= \langle\sigma|0\rangle = \frac{1}{\sqrt[4]{5}}\frac{s_1}{\sqrt{s_2}}, \\ \left\langle \sigma'\left|I(\zeta,\overline{\zeta})\right|0\right\rangle &= \langle\sigma'|0\rangle = \frac{1}{\sqrt[4]{5}}\sqrt{s_2}\end{aligned} \tag{50}$$

For the operator ε

$$\begin{aligned}
\left\langle I \left| \varepsilon(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= \left\langle I \left| \mathcal{V}_{1,2}(\zeta)\overline{\mathcal{V}}_{3,3}(\overline{\zeta}) \right| 0,0;\alpha_0 \right\rangle \\
&= \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_1}{2}}\left\langle B(\alpha_{1,2})\right| \mathcal{V}_{1,2}(\zeta)\overline{\mathcal{V}}_{3,3}(\overline{\zeta}) \left|0,0;\alpha_0\right\rangle \\
&= \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_1}{2}}(1-\zeta\overline{\zeta})^{-\frac{1}{5}}, \\
\left\langle \varepsilon \left| \varepsilon(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= -\frac{1}{\sqrt[4]{5}}\frac{s_2}{\sqrt{2s_1}}(1-\zeta\overline{\zeta})^{-\frac{1}{5}}, \\
\left\langle \varepsilon' \left| \varepsilon(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= -\frac{1}{\sqrt[4]{5}}\frac{s_2}{\sqrt{2s_1}}(1-\zeta\overline{\zeta})^{-\frac{1}{5}}, \\
\left\langle \varepsilon'' \left| \varepsilon(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= \frac{1}{\sqrt[4]{5}}\sqrt{\frac{s_1}{2}}(1-\zeta\overline{\zeta})^{-\frac{1}{5}}, \\
\left\langle \sigma \left| \varepsilon(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= \frac{1}{\sqrt[4]{5}}\frac{s_2}{\sqrt{s_1}}(1-\zeta\overline{\zeta})^{-\frac{1}{5}}, \\
\left\langle \sigma' \left| \varepsilon(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= -\frac{1}{\sqrt[4]{5}}\sqrt{s_1}(1-\zeta\overline{\zeta})^{-\frac{1}{5}}
\end{aligned} \tag{51}$$

and for the spin operator σ

$$\begin{aligned}
\left\langle I \left| \sigma(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= \frac{1}{\sqrt[4]{5}}s_1(1-\zeta\overline{\zeta})^{-\frac{3}{40}}, \\
\left\langle \varepsilon \left| \sigma(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= \frac{1}{\sqrt[4]{10}}\frac{s_2}{\sqrt{s_1}}(1-\zeta\overline{\zeta})^{-\frac{3}{40}}, \\
\left\langle \varepsilon' \left| \sigma(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= -\frac{1}{\sqrt[4]{10}}\frac{s_2}{\sqrt{s_1}}(1-\zeta\overline{\zeta})^{-\frac{3}{40}}, \\
\left\langle \varepsilon'' \left| \sigma(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= -\frac{1}{\sqrt[4]{10}}\sqrt{s_1}(1-\zeta\overline{\zeta})^{-\frac{3}{40}}, \\
\left\langle \sigma \left| \sigma(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= 0, \\
\left\langle \sigma' \left| \sigma(\zeta,\overline{\zeta}) \right| 0 \right\rangle &= 0
\end{aligned} \tag{52}$$

At this level one can come back to the UHP by the global conformal transformation

$$z = -iy_0\frac{\zeta-1}{\zeta+1} \qquad , \qquad \overline{z} = iy_0\frac{\overline{\zeta}-1}{\overline{\zeta}+1} \tag{53}$$

which map the origin $\zeta = 0$ to the point $z = iy_0$. The effect of these transformation on n-point correlation functions is given by

$$\begin{aligned}
< \ & \phi_1(z_1,\overline{z}_1)\phi_2(z_2,\overline{z}_2)......\phi_n(z_p,\overline{z}_p) >_{UHP} \\
= \ & \prod_{i=1}^{n}\left(\frac{dz_i}{d\zeta_i}\right)^{-h_i}\left(\frac{d\overline{z}_i}{d\overline{\zeta}_i}\right)^{-h_i} < \phi_1(\zeta_1,\overline{\zeta}_1)\phi_2(\zeta_2,\overline{\zeta}_2)......\phi_n(\zeta_p,\overline{\zeta}_p) >_{Disk} \\
= \ & \prod_{i=1}^{n}\left(\frac{2y_0}{(\zeta_i+1)(\overline{\zeta}_i+1)}\right)^{-2h_i} < \phi_1(\zeta_1,\overline{\zeta}_1)\phi_2(\zeta_2,\overline{\zeta}_2)......\phi_n(\zeta_p,\overline{\zeta}_p) >_{Disk}
\end{aligned}$$

For example the one point correlators given in equations (51) and (52)are transformed by the transformation (53) into

$$\begin{aligned}
\langle\varepsilon(z,\overline{z})\rangle_I &= \frac{\langle I\left|\varepsilon(z,\overline{z})\right|0,0;\alpha_0\rangle}{\langle I|0,0;\alpha_0\rangle} = \left(\frac{s_1}{s_2}\right)^{\frac{1}{2}}(2y)^{-\frac{1}{5}}, \\
\langle\varepsilon(z,\overline{z})\rangle_\varepsilon &= -\left(\frac{s_2}{s_1}\right)^{\frac{3}{2}}(2y)^{-\frac{1}{5}}, \\
\langle\varepsilon(z,\overline{z})\rangle_{\varepsilon'} &= -\left(\frac{s_2}{s_1}\right)^{\frac{3}{2}}(2y)^{-\frac{1}{5}}, \\
\langle\varepsilon(z,\overline{z})\rangle_{\varepsilon''} &= \left(\frac{s_1}{s_2}\right)^{\frac{1}{2}}(2y)^{-\frac{1}{5}}, \\
\langle\varepsilon(z,\overline{z})\rangle_\sigma &= \left(\frac{s_2}{s_1}\right)^{\frac{3}{2}}(2y)^{-\frac{1}{5}}, \\
\langle\varepsilon(z,\overline{z})\rangle_{\sigma'} &= -\left(\frac{s_1}{s_2}\right)^{\frac{1}{2}}(2y)^{-\frac{1}{5}}, \\
\langle\sigma(z,\overline{z})\rangle_I &= \sqrt{2}\frac{s_1}{\sqrt{s_2}}(2y)^{-\frac{3}{40}}, \\
\langle\sigma(z,\overline{z})\rangle_\varepsilon &= \sqrt[4]{2}\left(\frac{s_2}{s_1}\right)^{\frac{3}{2}}(2y)^{-\frac{3}{40}}, \\
\langle\sigma(z,\overline{z})\rangle_{\varepsilon'} &= -\sqrt[4]{2}\left(\frac{s_2}{s_1}\right)^{\frac{3}{2}}(2y)^{-\frac{3}{40}}, \\
\langle\sigma(z,\overline{z})\rangle_{\varepsilon''} &= -\sqrt[4]{2}\left(\frac{s_1}{s_2}\right)^{\frac{1}{2}}(2y)^{-\frac{3}{40}}, \\
\langle\sigma(z,\overline{z})\rangle_\sigma &= 0, \\
\langle\sigma(z,\overline{z})\rangle_{\sigma'} &= 0
\end{aligned} \tag{54}$$

5.3 The boundary 2-point boundary correlation functions:

We consider the two point function of the field $\Phi_{1,2} \equiv \varepsilon$. They are of the form

$$\begin{aligned} & \langle B(\alpha)|\, \Phi_{1,2}(\zeta_1, \overline{\zeta}_1)\Phi_{1,2}(\zeta_2, \overline{\zeta}_2)\, |0,0;\alpha_0\rangle \\ = \; & \langle B(\alpha)|\, \mathcal{V}_{1,2}^{m_1,n_1}(\zeta_1)\overline{\mathcal{V}}_{(p'-1,p-2)}^{\overline{m}_1,\overline{n}_1}(\overline{\zeta}_1)\mathcal{V}_{1,2}^{m_2,n_2}(\zeta_2)\overline{\mathcal{V}}_{1,2}^{\overline{m}_2,\overline{n}_2}(\overline{\zeta}_2)\, |0,0;\alpha_0\rangle \end{aligned} \quad (55)$$

where we taked first $\Phi_{1,2}(\zeta_1, \overline{\zeta}_1) \sim \mathcal{V}_{1,2}(\zeta_1)\overline{\mathcal{V}}_{(p'-1,p-2)}(\overline{\zeta}_1)$ and then $\Phi_{1,2}(\zeta_1, \overline{\zeta}_1) \sim \mathcal{V}_{1,2}(\zeta_2)\overline{\mathcal{V}}_{1,2}(\overline{\zeta}_2)$.

From the neutrallity condition, we obtain the following contraints

$$\begin{aligned} \alpha &= (m_1 + m_2)\alpha_+ + (n_1 + n_2 - 1)\alpha_-, \\ 2\alpha_0 - \alpha &= (\overline{m}_1 + \overline{m}_2 + 1)\alpha_+ + (\overline{n}_1 + \overline{n}_2 + 1)\alpha_- \end{aligned} \quad (56)$$

which can be solved as

$$m = \overline{m} = \overline{n} = 0, n = 1, \quad (57)$$

$$m = \overline{m} = n = 0, \overline{n} = 1 \quad (58)$$

with $m = m_1 + m_2, n = n_1 + n_2, \overline{m} = \overline{m}_1 + \overline{m}_2, \overline{n} = \overline{n}_1 + \overline{n}_2$.

The first solution (57) corresponds to $\alpha = \alpha_{1,1} = 0$, and to the conformal block

$$I_1 = N_1(1 - \zeta_1\overline{\zeta}_1)^a(1 - \zeta_2\overline{\zeta}_2)^a\,[\eta(\eta - 1)]^a \quad (59)$$

$$\times\frac{\Gamma(1 - \alpha_-^2)^2}{\Gamma(2 - 2\alpha_-^2)}F(2a, 1 - \alpha_-^2, 2 - 2\alpha_-^2; \eta) \quad (60)$$

with $a = 2\alpha_{1,2}(2\alpha_0 - \alpha_{1,2})$, F being the hypergeometric function and η is the parameter defined by

$$\eta = \frac{(\zeta_1 - \zeta_2)(\overline{\zeta}_1 - \overline{\zeta}_2)}{(1 - \zeta_1\overline{\zeta}_1)(1 - \zeta_2\overline{\zeta}_2)}$$

The second solution (58) gives $\alpha = \alpha_{1,3} = -\alpha_-$, and corresponds to the conformal block

$$\begin{aligned} I_2 = \; & N_2(1 - \zeta_1\overline{\zeta}_1)^a(1 - \zeta_2\overline{\zeta}_2)^a\,[\eta(\eta - 1)]^a\,(-\eta)^{b-a} \\ & \times\frac{\Gamma(1 - \alpha_-^2)\Gamma(3\alpha_-^2 - 1)}{\Gamma(2\alpha_-^2)}F(\alpha_-^2, 1 - \alpha_-^2, 2\alpha_-^2; \eta) \end{aligned} \quad (61)$$

with $b = 2\alpha_{1,2}^2$.

Using (53) one can write these equations on the UHP as

$$\begin{aligned} I_1 &= N_1 \left\{ \frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)} \right\}^{2h_{1,2}} \\ &\quad \times \frac{\Gamma(1-\alpha_-^2)^2}{\Gamma(2-2\alpha_-^2)} F(-4h_{1,2}, 1-\alpha_-^2, 2-2\alpha_-^2; \eta) \end{aligned} \tag{62}$$

and

$$\begin{aligned} I_2 &= N_2 \left\{ \frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)} \right\}^{2h_{1,2}} \\ &\quad \times \frac{\Gamma(1-\alpha_-^2)\Gamma(3\alpha_-^2-1)}{\Gamma(2\alpha_-^2)} F(\alpha_-^2, 1-\alpha_-^2, 2\alpha_-^2; \eta) \end{aligned} \tag{63}$$

with

$$\eta = \frac{(\zeta_1-\zeta_2)(\overline{\zeta}_1-\overline{\zeta}_2)}{(1-\zeta_1\overline{\zeta}_1)(1-\zeta_2\overline{\zeta}_2)} \rightarrow \frac{(z_1-z_2)(\overline{z}_1-\overline{z}_2)}{(z_1-\overline{z}_1)(z_2-\overline{z}_2)}$$

Combining the two blocks with the coefficients apparing in the development of the boundary states we obtain for the correlators in the UHP

$$\begin{aligned} \langle \varepsilon(z_1,\overline{z}_1)\varepsilon(z_2,\overline{z}_2)\rangle_{\widetilde{I}} &= \left\{ \frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)} \right\}^{\frac{1}{5}} \times \\ &\quad \left\{ N_1 \frac{\Gamma(1/5)^2}{\Gamma(2/5)} F_1 + \sqrt{\frac{s_1}{s_2}} N_2 \frac{\Gamma(\frac{1}{5})\Gamma(\frac{7}{5})}{\Gamma(\frac{8}{5})} \times (-\eta)^{9/5} F_2 \right\}, \end{aligned}$$

$$\begin{aligned} \langle \varepsilon(z_1,\overline{z}_1)\varepsilon(z_2,\overline{z}_2)\rangle_{\widetilde{\varepsilon}} &= \left\{ \frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)} \right\}^{\frac{1}{5}} \times \\ &\quad \left\{ N_1 \frac{\Gamma(1/5)^2}{\Gamma(2/5)} F_1 - \left(\frac{s_2}{s_1}\right)^{3/2} N_2 \frac{\Gamma(\frac{1}{5})\Gamma(\frac{7}{5})}{\Gamma(\frac{8}{5})} \times (-\eta)^{9/5} F_2 \right\}, \end{aligned}$$

$$\begin{aligned} \langle \varepsilon(z_1,\overline{z}_1)\varepsilon(z_2,\overline{z}_2)\rangle_{\widetilde{\varepsilon}'} &= \left\{ \frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)} \right\}^{\frac{1}{5}} \times \\ &\quad \left\{ N_1 \frac{\Gamma(1/5)^2}{\Gamma(2/5)} F_1 - \left(\frac{s_2}{s_1}\right)^{3/2} N_2 \frac{\Gamma(\frac{1}{5})\Gamma(\frac{7}{5})}{\Gamma(\frac{8}{5})} \times (-\eta)^{9/5} F_2 \right\}, \end{aligned}$$

$$
\begin{aligned}
\langle\varepsilon(z_1,\overline{z}_1)\varepsilon(z_2,\overline{z}_2)\rangle_{\widetilde{\varepsilon}''} &= \left\{\frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)}\right\}^{\frac{1}{5}}\times \\
&\left\{N_1\frac{\Gamma(1/5)^2}{\Gamma(2/5)}F_1+\left(\frac{s_1}{s_2}\right)^{1/2}N_2\frac{\Gamma(\frac{1}{5})\Gamma(\frac{7}{5})}{\Gamma(\frac{8}{5})}\times(-\eta)^{9/5}F_2\right\}, \\
\langle\varepsilon(z_1,\overline{z}_1)\varepsilon(z_2,\overline{z}_2)\rangle_{\widetilde{\sigma}} &= \left\{\frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)}\right\}^{\frac{1}{5}}\times \\
&\left\{N_1\frac{\Gamma(1/5)^2}{\Gamma(2/5)}F_1-\left(\frac{s_2}{s_1}\right)^{3/2}N_2\frac{\Gamma(\frac{1}{5})\Gamma(\frac{7}{5})}{\Gamma(\frac{8}{5})}\times(-\eta)^{9/5}F_2\right\}, \\
\langle\varepsilon(z_1,\overline{z}_1)\varepsilon(z_2,\overline{z}_2)\rangle_{\widetilde{\sigma}'} &= \left\{\frac{(z_1-\overline{z}_1)(\overline{z}_2-z_2)}{(z_1-z_2)(\overline{z}_1-\overline{z}_2)(z_1-\overline{z}_2)(\overline{z}_1-z_2)}\right\}^{\frac{1}{5}}\times \\
&\left\{N_1\frac{\Gamma(1/5)^2}{\Gamma(2/5)}F_1+\left(\frac{s_1}{s_2}\right)^{1/2}N_2\frac{\Gamma(\frac{1}{5})\Gamma(\frac{7}{5})}{\Gamma(\frac{8}{5})}\times(-\eta)^{9/5}F_2\right\}
\end{aligned}
\tag{64}
$$

tel que $F_1 = F(-\frac{4}{10},\frac{1}{5},\frac{2}{5};\eta)$, et $F_2 = F(\frac{4}{5},\frac{1}{5},\frac{8}{5};\eta)$.

6 Conclusion

In this paper we have obtained the boundary consistent states and correlation functions of the tricritical Ising model, using the Coulomb-gas formalism. The results obtained for 1-point correlators are in concordance with that already known (see for example chapter 15 of [19]). We expect that an interpretation of the boundary states could be obtained from simulation methods.

We hope also to generalize the use of this method to the case of the nondiagonal minimal models, such as the one representing the tricritical 3-states Potts model.

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